

# VCE Mathematical Methods Formula Sheet Explained

An annotated guide to the formulas, variables, common exam uses and mistakes on the Mathematical Methods formula sheet.

## How to use this guide

The formula sheet is most useful when you already recognise the question type. Use this resource to learn what each formula is doing, what every symbol means and what to check before substituting values. The aim is to turn the official formula sheet into a fast exam reference rather than something you need to decode under time pressure.

## Formula sheet sections

|                    |                                                                                                               |
|--------------------|---------------------------------------------------------------------------------------------------------------|
| Mensuration        | Areas, surface areas and volumes used in geometry and application questions.                                  |
| Calculus           | Differentiation, integration, product/quotient/chain rules, Newton's method and trapezium-rule approximation. |
| Probability        | Probability rules, expectation, variance and discrete/binomial/continuous distributions.                      |
| Sample proportions | Sample proportion, standard deviation and approximate confidence intervals.                                   |

# 1. Mensuration

These formulas are usually straightforward, but marks are often lost by choosing the wrong dimension, mixing radius and diameter, or forgetting whether a question asks for area, surface area or volume.

## Area of a trapezium

$$A = \frac{1}{2}(a + b)h$$

What it means: Finds the area of a trapezium using the two parallel side lengths and the perpendicular height.

Variables:  $A$  = area;  $a$  and  $b$  = parallel sides;  $h$  = perpendicular height.

When to use it: When a region is trapezoidal or can be broken into a trapezium.

Watch out for: The height must be perpendicular to the parallel sides.

Quick example: If  $a = 8$ ,  $b = 12$  and  $h = 5$ , then  $A = 50$ .

## Curved surface area of a cylinder

$$A = 2\pi rh$$

What it means: Finds only the curved side of a cylinder, excluding the two circular ends.

Variables:  $r$  = radius;  $h$  = height.

When to use it: Wrapping, labels and lateral surface-area questions.

Watch out for: Do not add  $2\pi r^2$  unless the circular ends are required.

## Volume of a cylinder

$$V = \pi r^2 h$$

What it means: Finds the volume enclosed by a cylinder.

Variables:  $r$  = radius;  $h$  = perpendicular height.

When to use it: Tanks, pipes, containers and circular-cross-section solids.

Watch out for: Check whether a diameter has been supplied; radius is half the diameter.

## Volume of a cone

$$V = \frac{1}{3} \pi r^2 h$$

What it means: A cone has one-third the volume of a cylinder with the same base and height.

Variables:  $r$  = base radius;  $h$  = perpendicular height.

When to use it: Cone-shaped solids and composite-volume questions.

Watch out for: Use perpendicular height, not slant height.

## Volume of a pyramid

$$V = \frac{1}{3} Ah$$

What it means: Finds the volume of a pyramid from the area of its base.

Variables: A = base area; h = perpendicular height.

When to use it: Any pyramid where the base area can first be calculated.

Watch out for: A is base area, not total surface area.

### Volume of a sphere

$$V = \frac{4}{3} \pi r^3$$

What it means: Finds the volume of a sphere.

Variables: r = radius.

When to use it: Spherical or approximately spherical objects.

Watch out for: The radius is cubed, so unit-conversion errors can become large.

### Area of a triangle

$$A = \frac{1}{2} bc \sin(A)$$

What it means: Finds triangle area when two side lengths and their included angle are known.

Variables: b and c = two sides; A = angle between them.

When to use it: Non-right-angled triangles without direct base-height information.

Watch out for: The angle must be the included angle between b and c.

## 2. Calculus - Differentiation

Differentiation formulas describe rates of change and gradients. The main challenge is usually recognising the function structure before choosing the rule.

### Power rule

$$\frac{d}{dx} (x^n) = n x^{n-1}$$

What it means: Differentiates a power of  $x$  by multiplying by the exponent and reducing the exponent by 1.

Variables:  $n$  = real exponent.

When to use it: Polynomial and power functions.

Watch out for: Rewrite roots and reciprocal powers as indices first where helpful.

### Linear expression to a power

$$\frac{d}{dx} ((ax+b)^n) = an(ax+b)^{n-1}$$

What it means: Combines the power rule with the chain rule.

Variables:  $a, b$  = constants;  $n$  = exponent.

When to use it: Powers of linear expressions such as  $(3x-2)^5$ .

Watch out for: The extra factor  $a$  comes from differentiating the inside expression.

### Exponential function

$$\frac{d}{dx} (e^{(ax)}) = a e^{(ax)}$$

What it means: Differentiates an exponential base- $e$  function.

Variables:  $a$  = constant.

When to use it: Growth/decay models and exponential functions.

Watch out for: Keep the original exponential and multiply by the derivative of the exponent.

### Natural logarithm

$$\frac{d}{dx} (\ln x) = 1/x$$

What it means: Differentiates the natural logarithm.

Variables:  $x > 0$  for  $\ln(x)$  in the real-number setting.

When to use it: Logarithmic functions and chain-rule forms such as  $\ln(g(x))$ .

Watch out for: For  $\ln(g(x))$ , use  $g'(x)/g(x)$ .

### Sine

$$\frac{d}{dx} (\sin(ax)) = a \cos(ax)$$

What it means: Differentiates a sine function.

Variables:  $a$  = coefficient of  $x$  inside the angle.

When to use it: Circular-function differentiation.

Watch out for: Calculus with circular functions uses radians.

## Cosine

$$\frac{d}{dx}(\cos(ax)) = -a \sin(ax)$$

What it means: Differentiates a cosine function.

Variables:  $a$  = coefficient of  $x$ .

When to use it: Circular-function differentiation.

Watch out for: Do not forget the negative sign.

## Tangent

$$\frac{d}{dx}(\tan(ax)) = a \sec^2(ax)$$

What it means: Differentiates a tangent function.

Variables:  $a$  = coefficient of  $x$ .

When to use it: Circular-function differentiation.

Watch out for: Equivalent form:  $a/\cos^2(ax)$ .

## Product rule

$$\frac{d}{dx}(uv) = u(\frac{dv}{dx}) + v(\frac{du}{dx})$$

What it means: Differentiates a product of two functions.

Variables:  $u$  and  $v$  = functions of  $x$ .

When to use it: When two non-trivial functions are multiplied.

Watch out for: Do not multiply the two derivatives together.

## Quotient rule

$$\frac{d}{dx}(u/v) = [v(\frac{du}{dx}) - u(\frac{dv}{dx})] / v^2$$

What it means: Differentiates one function divided by another.

Variables:  $u$  = numerator;  $v$  = denominator.

When to use it: When a quotient cannot be simplified more easily first.

Watch out for: The denominator is  $v^2$  and the subtraction order matters.

## Chain rule

$$\frac{dy}{dx} = \left(\frac{dy}{du}\right)\left(\frac{du}{dx}\right)$$

What it means: Differentiates a composite function by differentiating the outside and multiplying by the derivative of the inside.

Variables:  $u$  = intermediate or inner function.

When to use it: Functions inside functions such as  $(\sin x)^4$  or  $e^{(x^2)}$ .

Watch out for: Look for nested structure before differentiating.

### 3. Calculus - Integration and Numerical Methods

Integration reverses differentiation and is used for antiderivatives, signed area and accumulated change. Numerical methods are used when an exact algebraic solution is inconvenient or unavailable.

#### Power antiderivative

$$x^n dx = x^{n+1}/(n+1) + C, n \neq -1$$

What it means: Reverses the power rule.

Variables:  $n$  = exponent;  $C$  = constant of integration.

When to use it: Indefinite integration of powers.

Watch out for: The rule does not apply to  $n = -1$ .

#### Linear expression power

$$(ax+b)^n dx = (ax+b)^{n+1}/[a(n+1)] + C$$

What it means: Antidifferentiates a power of a linear expression.

Variables:  $a, b$  = constants;  $n \neq -1$ .

When to use it: Expressions such as  $(3x+1)^4$ .

Watch out for: Divide by the inside coefficient  $a$ .

#### Exponential integral

$$e^{ax} dx = (1/a)e^{ax} + C$$

What it means: Antidifferentiates an exponential base- $e$  function.

Variables:  $a$  = non-zero constant.

When to use it: Exponential functions.

Watch out for: The factor  $1/a$  compensates for the derivative of  $ax$ .

#### Reciprocal integral

$$1/x dx = \ln(x) + C, x > 0$$

What it means: Gives the antiderivative of  $1/x$ .

Variables:  $x$  = variable.

When to use it: The special  $n = -1$  case.

Watch out for: This is the positive- $x$  form shown on the sheet.

#### Sine integral

$$\sin(ax) dx = -(1/a)\cos(ax) + C$$

What it means: Antidifferentiates sine.

Variables:  $a$  = coefficient of  $x$ .

When to use it: Circular-function integration.

Watch out for: Do not forget the negative sign.

### Cosine integral

$$\cos(ax) \, dx = (1/a)\sin(ax) + C$$

What it means: Antidifferentiates cosine.

Variables:  $a$  = coefficient of  $x$ .

When to use it: Circular-function integration.

Watch out for: Divide by the inside coefficient  $a$ .

### Newton's method

$$x_{n+1} = x_n - f(x_n)/f'(x_n)$$

What it means: Uses tangent-line approximations to iteratively estimate a solution to  $f(x)=0$ .

Variables:  $x_n$  = current estimate;  $x_{n+1}$  = next estimate.

When to use it: Approximating roots or  $x$ -intercepts.

Watch out for: A poor starting value or derivative near zero can cause poor convergence.

Quick example: Start at  $x_0$ , compute  $f(x_0)$  and  $f'(x_0)$ , then find  $x_1$  and repeat.

### Trapezium rule

$$\text{Area} \approx (x_n - x_0)/(2n) [f(x_0) + 2f(x_1) + \dots + 2f(x_{n-1}) + f(x_n)]$$

What it means: Approximates area under a curve using  $n$  equal-width trapeziums.

Variables:  $x_0$  and  $x_n$  = endpoints;  $n$  = number of equal subintervals.

When to use it: Numerical area approximation.

Watch out for: Only the first and last ordinates have coefficient 1; interior ordinates have coefficient 2.

## 4. Probability Rules

Probability formulas are short, but Methods questions often test whether you understand the relationship between events rather than simply substitute numbers.

### Complement rule

$$\Pr(A) = 1 - \Pr(A^c)$$

What it means: An event and its complement together have probability 1.

Variables:  $A^c$  = 'not A'.

When to use it: Questions involving 'at least one', 'none' or 'not'.

Watch out for: Define the complement event carefully.

### Addition rule

$$\Pr(A \cup B) = \Pr(A) + \Pr(B) - \Pr(AB)$$

What it means: Finds the probability that A or B, or both, occur.

Variables:  $A \cup B$  = union;  $AB$  = intersection.

When to use it: Any two-event 'A or B' probability.

Watch out for: Subtract the overlap once because it was counted twice.

### Conditional probability

$$\Pr(A | B) = \Pr(AB) / \Pr(B)$$

What it means: Finds the probability of A given that B has occurred.

Variables:  $\Pr(B)$  must be non-zero.

When to use it: When known information restricts the sample space.

Watch out for: The denominator is the conditioning event B.

### Mean / expected value

$$\mu = E(X)$$

What it means: Represents the long-run average value of a random variable.

Variables: X = random variable;  $\mu$  = mean.

When to use it: Probability distributions and expected-value questions.

Watch out for: Expected value does not need to be a possible observed value.

### Variance

$$\text{var}(X) = \sigma^2 = E((X-\mu)^2) = E(X^2) - \mu^2$$

What it means: Measures spread around the mean.

Variables:  $\sigma^2$  = variance;  $\sigma$  = standard deviation;  $\mu$  = mean.

When to use it: Comparing spread or calculating standard deviation.

Watch out for: Variance is in squared units; standard deviation is variance.

### Binomial coefficient

$$C(n,x) = \frac{n!}{x!(n-x)!}$$

What it means: Counts the number of ways to choose x successes from n trials.

Variables: n = number of trials; x = number selected.

When to use it: Binomial probability and counting.

Watch out for: Order does not matter.

## 5. Probability Distributions

The formula sheet separates discrete, binomial and continuous distributions. First identify which type of random variable you are dealing with.

### Discrete distribution

$$\Pr(X=x) = p(x)$$

What it means: Assigns a probability to each possible discrete value of X.

Variables:  $p(x)$  = probability mass function.

When to use it: Finite or countable outcome sets.

Watch out for: Probabilities must be non-negative and sum to 1.

### Discrete mean

$$\mu = \sum x p(x)$$

What it means: Calculates the expected value of a discrete random variable.

Variables:  $x$  = possible value;  $p(x)$  = its probability.

When to use it: Probability tables and discrete distributions.

Watch out for: Multiply each value by its own probability before summing.

### Discrete variance

$$\sigma^2 = \sum (x-\mu)^2 p(x)$$

What it means: Calculates spread for a discrete distribution.

Variables:  $\mu$  = distribution mean.

When to use it: Discrete probability tables.

Watch out for: An equivalent shortcut is  $E(X^2) - [E(X)]^2$ .

### Binomial probability

$$\Pr(X=x) = C(n,x)p^x(1-p)^{n-x}$$

What it means: Finds the probability of exactly  $x$  successes in  $n$  independent Bernoulli trials.

Variables:  $n$  = trials;  $x$  = successes;  $p$  = success probability.

When to use it: Repeated independent yes/no trials with constant  $p$ .

Watch out for: Check the binomial conditions before using the model.

### Binomial mean and variance

$$\mu = np \text{ and } \sigma^2 = np(1-p)$$

What it means: Gives the centre and spread of a binomial random variable.

Variables:  $n$  = trials;  $p$  = success probability.

When to use it: Summary statistics for  $X \sim \text{Bin}(n, p)$ .

Watch out for: Standard deviation is  $[\text{np}(1-p)]$ , not  $\text{np}(1-p)$ .

### Continuous probability

$$\Pr(a < X < b) = \text{from } a \text{ to } b f(x) dx$$

What it means: Probability is the area under the density function between  $a$  and  $b$ .

Variables:  $f(x)$  = probability density function.

When to use it: Continuous random variables.

Watch out for: For a continuous variable,  $\Pr(X=a)=0$ .

### Continuous mean

$$\mu = \int x f(x) dx \text{ over the full domain}$$

What it means: Calculates the expected value of a continuous random variable.

Variables:  $f(x)$  = density function.

When to use it: Continuous-distribution expected value.

Watch out for: Use the correct support/domain limits.

### Continuous variance

$$\sigma^2 = \int (x-\mu)^2 f(x) dx$$

What it means: Measures spread of a continuous random variable.

Variables:  $\mu$  = mean.

When to use it: Continuous-distribution variance.

Watch out for: The shortcut  $E(X^2) - [E(X)]^2$  may be easier.

## 6. Sample Proportions

These formulas connect a sample proportion to the underlying population proportion and are central to confidence-interval questions.

### Sample proportion

$$\hat{p} = X/n$$

What it means: The proportion of sampled observations classified as successes.

Variables:  $X$  = number of successes;  $n$  = sample size;  $\hat{p}$  = sample proportion.

When to use it: Whenever sample data are reported as a count out of  $n$ .

Watch out for: Distinguish  $\hat{p}$ , the sample statistic, from  $p$ , the population parameter.

### Mean of sample proportion

$$E(\hat{p}) = p$$

What it means: Shows that the sample proportion is an unbiased estimator of the population proportion.

Variables:  $p$  = true population proportion.

When to use it: Sampling-distribution questions.

Watch out for:  $p$  is typically unknown in real estimation problems.

### Standard deviation of sample proportion

$$sd(\hat{p}) = [p(1-p)/n]$$

What it means: Measures the typical sampling variation in  $\hat{p}$ .

Variables:  $p$  = population proportion;  $n$  = sample size.

When to use it: Sampling distributions when  $p$  is known.

Watch out for: Larger  $n$  reduces the standard deviation.

### Approximate confidence interval

$$\hat{p} \pm z[\hat{p}(1-\hat{p})/n]$$

What it means: Builds an approximate interval estimate for a population proportion.

Variables:  $\hat{p}$  = sample proportion;  $n$  = sample size;  $z$  = critical value.

When to use it: Confidence intervals for population proportions.

Watch out for: You still need the appropriate  $z$ -value and a contextual interpretation.

Quick example: For a 95% interval,  $z$  is commonly 1.96.