



International Baccalaureate®  
Baccalauréat International  
Bachillerato Internacional

Diploma Programme

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# Mathematics: analysis and approaches formula booklet

For use during the course and in the examinations  
First examinations 2021

Version 1.3



## Contents

### Prior learning

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|           |   |
|-----------|---|
| SL and HL | 2 |
|-----------|---|

### Topic 1: Number and algebra

---

|           |   |
|-----------|---|
| SL and HL | 3 |
|-----------|---|

|         |   |
|---------|---|
| HL only | 4 |
|---------|---|

### Topic 2: Functions

---

|           |   |
|-----------|---|
| SL and HL | 5 |
|-----------|---|

|         |   |
|---------|---|
| HL only | 5 |
|---------|---|

### Topic 3: Geometry and trigonometry

---

|           |   |
|-----------|---|
| SL and HL | 6 |
|-----------|---|

|         |   |
|---------|---|
| HL only | 7 |
|---------|---|

### Topic 4: Statistics and probability

---

|           |   |
|-----------|---|
| SL and HL | 9 |
|-----------|---|

|         |    |
|---------|----|
| HL only | 10 |
|---------|----|

### Topic 5: Calculus

---

|           |    |
|-----------|----|
| SL and HL | 11 |
|-----------|----|

|         |    |
|---------|----|
| HL only | 12 |
|---------|----|

## Prior learning – SL and HL

|                                                                                            |                                                                                         |
|--------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------|
| Area of a parallelogram                                                                    | $A = bh$ , where $b$ is the base, $h$ is the height                                     |
| Area of a triangle                                                                         | $A = \frac{1}{2}(bh)$ , where $b$ is the base, $h$ is the height                        |
| Area of a trapezoid                                                                        | $A = \frac{1}{2}(a + b)h$ , where $a$ and $b$ are the parallel sides, $h$ is the height |
| Area of a circle                                                                           | $A = \pi r^2$ , where $r$ is the radius                                                 |
| Circumference of a circle                                                                  | $C = 2\pi r$ , where $r$ is the radius                                                  |
| Volume of a cuboid                                                                         | $V = lwh$ , where $l$ is the length, $w$ is the width, $h$ is the height                |
| Volume of a cylinder                                                                       | $V = \pi r^2 h$ , where $r$ is the radius, $h$ is the height                            |
| Volume of a prism                                                                          | $V = Ah$ , where $A$ is the area of cross-section, $h$ is the height                    |
| Area of the curved surface of a cylinder                                                   | $A = 2\pi rh$ , where $r$ is the radius, $h$ is the height                              |
| Distance between two points $(x_1, y_1)$ and $(x_2, y_2)$                                  | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2}$                                              |
| Coordinates of the midpoint of a line segment with endpoints $(x_1, y_1)$ and $(x_2, y_2)$ | $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$                               |

## Topic 1: Number and algebra – SL and HL

|               |                                                                                                                                |                                                                                                                                                                                                                                                                                                                           |
|---------------|--------------------------------------------------------------------------------------------------------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>SL 1.2</b> | <p>The <math>n</math>th term of an arithmetic sequence</p> <p>The sum of <math>n</math> terms of an arithmetic sequence</p>    | $u_n = u_1 + (n-1)d$ $S_n = \frac{n}{2}(2u_1 + (n-1)d); S_n = \frac{n}{2}(u_1 + u_n)$                                                                                                                                                                                                                                     |
| <b>SL 1.3</b> | <p>The <math>n</math>th term of a geometric sequence</p> <p>The sum of <math>n</math> terms of a finite geometric sequence</p> | $u_n = u_1 r^{n-1}$ $S_n = \frac{u_1(r^n - 1)}{r - 1} = \frac{u_1(1 - r^n)}{1 - r}, r \neq 1$                                                                                                                                                                                                                             |
| <b>SL 1.4</b> | Compound interest                                                                                                              | $FV = PV \times \left(1 + \frac{r}{100k}\right)^{kn}$ <p>where <math>FV</math> is the future value,<br/> <math>PV</math> is the present value, <math>n</math> is the number of years,<br/> <math>k</math> is the number of compounding periods per year,<br/> <math>r\%</math> is the nominal annual rate of interest</p> |
| <b>SL 1.5</b> | Exponents and logarithms                                                                                                       | $a^x = b \Leftrightarrow x = \log_a b, \text{ where } a > 0, b > 0, a \neq 1$                                                                                                                                                                                                                                             |
| <b>SL 1.7</b> | Exponents and logarithms                                                                                                       | $\log_a xy = \log_a x + \log_a y$ $\log_a \frac{x}{y} = \log_a x - \log_a y$ $\log_a x^m = m \log_a x$ $\log_a x = \frac{\log_b x}{\log_b a}$                                                                                                                                                                             |
| <b>SL 1.8</b> | The sum of an infinite geometric sequence                                                                                      | $S_\infty = \frac{u_1}{1-r},  r  < 1$                                                                                                                                                                                                                                                                                     |
| <b>SL 1.9</b> | Binomial theorem $n \in \mathbb{N}$                                                                                            | $(a+b)^n = a^n + {}^nC_1 a^{n-1}b + \dots + {}^nC_r a^{n-r}b^r + \dots + b^n$ ${}^nC_r = \frac{n!}{r!(n-r)!}$                                                                                                                                                                                                             |

## Topic 1: Number and algebra – HL only

|                     |                                                       |                                                                                                                                |
|---------------------|-------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------|
| <b>AHL<br/>1.10</b> | Combinations                                          | ${}^nC_r = \frac{n!}{r!(n-r)!}$                                                                                                |
|                     | Permutations                                          | ${}^nP_r = \frac{n!}{(n-r)!}$                                                                                                  |
|                     | Extension of binomial theorem, $n \in \mathbb{Q}$     | $(a+b)^n = a^n \left( 1 + n\left(\frac{b}{a}\right) + \frac{n(n-1)}{2!}\left(\frac{b}{a}\right)^2 + \dots \right)$             |
| <b>AHL<br/>1.12</b> | Complex numbers                                       | $z = a + bi$                                                                                                                   |
| <b>AHL<br/>1.13</b> | Modulus-argument (polar) and exponential (Euler) form | $z = r(\cos \theta + i \sin \theta) = r e^{i\theta} = r \operatorname{cis} \theta$                                             |
| <b>AHL<br/>1.14</b> | De Moivre's theorem                                   | $[r(\cos \theta + i \sin \theta)]^n = r^n (\cos n\theta + i \sin n\theta) = r^n e^{in\theta} = r^n \operatorname{cis} n\theta$ |

## Topic 2: Functions – SL and HL

|                   |                                                       |                                                                                  |
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| <b>SL<br/>2.1</b> | Equations of a straight line                          | $y = mx + c$ ; $ax + by + d = 0$ ; $y - y_1 = m(x - x_1)$                        |
|                   | Gradient formula                                      | $m = \frac{y_2 - y_1}{x_2 - x_1}$                                                |
| <b>SL<br/>2.6</b> | Axis of symmetry of the graph of a quadratic function | $f(x) = ax^2 + bx + c \Rightarrow$ axis of symmetry is $x = -\frac{b}{2a}$       |
| <b>SL<br/>2.7</b> | Solutions of a quadratic equation                     | $ax^2 + bx + c = 0 \Rightarrow x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}, a \neq 0$ |
|                   | Discriminant                                          | $\Delta = b^2 - 4ac$                                                             |
| <b>SL<br/>2.9</b> | Exponential and logarithmic functions                 | $a^x = e^{x \ln a}$ ; $\log_a a^x = x = a^{\log_a x}$ where $a, x > 0, a \neq 1$ |

## Topic 2: Functions – HL only

|                     |                                                                                                |                                                                     |
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| <b>AHL<br/>2.12</b> | Sum and product of the roots of polynomial equations of the form<br>$\sum_{r=0}^n a_r x^r = 0$ | Sum is $-\frac{a_{n-1}}{a_n}$ ; product is $\frac{(-1)^n a_0}{a_n}$ |
|---------------------|------------------------------------------------------------------------------------------------|---------------------------------------------------------------------|

## Topic 3: Geometry and trigonometry – SL and HL

|                   |                                                                                                                                                                                                                                                                                                                                                                                                         |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |
|-------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>SL<br/>3.1</b> | <p>Distance between two points <math>(x_1, y_1, z_1)</math> and <math>(x_2, y_2, z_2)</math></p> <p>Coordinates of the midpoint of a line segment with endpoints <math>(x_1, y_1, z_1)</math> and <math>(x_2, y_2, z_2)</math></p> <p>Volume of a right-pyramid</p> <p>Volume of a right cone</p> <p>Area of the curved surface of a cone</p> <p>Volume of a sphere</p> <p>Surface area of a sphere</p> | $d = \sqrt{(x_1 - x_2)^2 + (y_1 - y_2)^2 + (z_1 - z_2)^2}$ $\left( \frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}, \frac{z_1 + z_2}{2} \right)$ $V = \frac{1}{3}Ah, \text{ where } A \text{ is the area of the base, } h \text{ is the height}$ $V = \frac{1}{3}\pi r^2 h, \text{ where } r \text{ is the radius, } h \text{ is the height}$ $A = \pi r l, \text{ where } r \text{ is the radius, } l \text{ is the slant height}$ $V = \frac{4}{3}\pi r^3, \text{ where } r \text{ is the radius}$ $A = 4\pi r^2, \text{ where } r \text{ is the radius}$ |
| <b>SL<br/>3.2</b> | <p>Sine rule</p> <p>Cosine rule</p> <p>Area of a triangle</p>                                                                                                                                                                                                                                                                                                                                           | $\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$ $c^2 = a^2 + b^2 - 2ab \cos C; \cos C = \frac{a^2 + b^2 - c^2}{2ab}$ $A = \frac{1}{2}ab \sin C$                                                                                                                                                                                                                                                                                                                                                                                             |
| <b>SL<br/>3.4</b> | <p>Length of an arc</p> <p>Area of a sector</p>                                                                                                                                                                                                                                                                                                                                                         | $l = r\theta, \text{ where } r \text{ is the radius, } \theta \text{ is the angle measured in radians}$ $A = \frac{1}{2}r^2\theta, \text{ where } r \text{ is the radius, } \theta \text{ is the angle measured in radians}$                                                                                                                                                                                                                                                                                                                         |

|                   |                            |                                                                                                                                          |
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| <b>SL<br/>3.5</b> | Identity for $\tan \theta$ | $\tan \theta = \frac{\sin \theta}{\cos \theta}$                                                                                          |
| <b>SL<br/>3.6</b> | Pythagorean identity       | $\cos^2 \theta + \sin^2 \theta = 1$                                                                                                      |
|                   | Double angle identities    | $\sin 2\theta = 2 \sin \theta \cos \theta$<br>$\cos 2\theta = \cos^2 \theta - \sin^2 \theta = 2 \cos^2 \theta - 1 = 1 - 2 \sin^2 \theta$ |

## Topic 3: Geometry and trigonometry – HL only

|                     |                                     |                                                                                                                                                                           |
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| <b>AHL<br/>3.9</b>  | Reciprocal trigonometric identities | $\sec \theta = \frac{1}{\cos \theta}$<br>$\operatorname{cosec} \theta = \frac{1}{\sin \theta}$                                                                            |
|                     | Pythagorean identities              | $1 + \tan^2 \theta = \sec^2 \theta$<br>$1 + \cot^2 \theta = \operatorname{cosec}^2 \theta$                                                                                |
| <b>AHL<br/>3.10</b> | Compound angle identities           | $\sin(A \pm B) = \sin A \cos B \pm \cos A \sin B$<br>$\cos(A \pm B) = \cos A \cos B \mp \sin A \sin B$<br>$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \mp \tan A \tan B}$ |
|                     | Double angle identity for $\tan$    | $\tan 2\theta = \frac{2 \tan \theta}{1 - \tan^2 \theta}$                                                                                                                  |
| <b>AHL<br/>3.12</b> | Magnitude of a vector               | $ \mathbf{v}  = \sqrt{v_1^2 + v_2^2 + v_3^2}$ , where $\mathbf{v} = \begin{pmatrix} v_1 \\ v_2 \\ v_3 \end{pmatrix}$                                                      |



## Topic 4: Statistics and probability – SL and HL

|                    |                                                  |                                                                           |
|--------------------|--------------------------------------------------|---------------------------------------------------------------------------|
| <b>SL<br/>4.2</b>  | Interquartile range                              | $IQR = Q_3 - Q_1$                                                         |
| <b>SL<br/>4.3</b>  | Mean, $\bar{x}$ , of a set of data               | $\bar{x} = \frac{\sum_{i=1}^k f_i x_i}{n}$ , where $n = \sum_{i=1}^k f_i$ |
| <b>SL<br/>4.5</b>  | Probability of an event $A$                      | $P(A) = \frac{n(A)}{n(U)}$                                                |
|                    | Complementary events                             | $P(A) + P(A') = 1$                                                        |
| <b>SL<br/>4.6</b>  | Combined events                                  | $P(A \cup B) = P(A) + P(B) - P(A \cap B)$                                 |
|                    | Mutually exclusive events                        | $P(A \cup B) = P(A) + P(B)$                                               |
|                    | Conditional probability                          | $P(A B) = \frac{P(A \cap B)}{P(B)}$                                       |
|                    | Independent events                               | $P(A \cap B) = P(A) P(B)$                                                 |
| <b>SL<br/>4.7</b>  | Expected value of a discrete random variable $X$ | $E(X) = \sum x P(X = x)$                                                  |
| <b>SL<br/>4.8</b>  | Binomial distribution<br>$X \sim B(n, p)$        |                                                                           |
|                    | Mean                                             | $E(X) = np$                                                               |
|                    | Variance                                         | $\text{Var}(X) = np(1 - p)$                                               |
| <b>SL<br/>4.12</b> | Standardized normal variable                     | $z = \frac{x - \mu}{\sigma}$                                              |

## Topic 4: Statistics and probability – HL only

|                     |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                      |                                                                                                                                                 |
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| <b>AHL<br/>4.13</b> | Bayes' theorem                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                       | $P(B A) = \frac{P(B) P(A B)}{P(B) P(A B) + P(B') P(A B')}$ $P(B_i A) = \frac{P(B_i)P(A B_i)}{P(B_1)P(A B_1) + P(B_2)P(A B_2) + P(B_3)P(A B_3)}$ |
| <b>AHL<br/>4.14</b> | <p>Variance <math>\sigma^2</math></p> $\sigma^2 = \frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n} = \frac{\sum_{i=1}^k f_i x_i^2}{n} - \mu^2$ <p>Standard deviation <math>\sigma</math></p> $\sigma = \sqrt{\frac{\sum_{i=1}^k f_i (x_i - \mu)^2}{n}}$ <p>Linear transformation of a single random variable</p> $E(aX + b) = aE(X) + b$ $\text{Var}(aX + b) = a^2 \text{Var}(X)$ <p>Expected value of a continuous random variable <math>X</math></p> $E(X) = \mu = \int_{-\infty}^{\infty} x f(x) dx$ <p>Variance</p> $\text{Var}(X) = E[(X - \mu)^2] = E(X^2) - [E(X)]^2$ <p>Variance of a discrete random variable <math>X</math></p> $\text{Var}(X) = \sum (x - \mu)^2 P(X = x) = \sum x^2 P(X = x) - \mu^2$ <p>Variance of a continuous random variable <math>X</math></p> $\text{Var}(X) = \int_{-\infty}^{\infty} (x - \mu)^2 f(x) dx = \int_{-\infty}^{\infty} x^2 f(x) dx - \mu^2$ |                                                                                                                                                 |

## Topic 5: Calculus – SL and HL

|               |                                                                                                                                                                                                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                  |
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| <b>SL 5.3</b> | Derivative of $x^n$                                                                                                                                                                                                        | $f(x) = x^n \Rightarrow f'(x) = nx^{n-1}$                                                                                                                                                                                                                                                                                                                                                                                                        |
| <b>SL 5.5</b> | <p>Integral of <math>x^n</math></p> <p>Area between a curve <math>y = f(x)</math> and the <math>x</math>-axis, where <math>f(x) &gt; 0</math></p>                                                                          | $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$ $A = \int_a^b y dx$                                                                                                                                                                                                                                                                                                                                                                           |
| <b>SL 5.6</b> | <p>Derivative of <math>\sin x</math></p> <p>Derivative of <math>\cos x</math></p> <p>Derivative of <math>e^x</math></p> <p>Derivative of <math>\ln x</math></p> <p>Chain rule</p> <p>Product rule</p> <p>Quotient rule</p> | $f(x) = \sin x \Rightarrow f'(x) = \cos x$ $f(x) = \cos x \Rightarrow f'(x) = -\sin x$ $f(x) = e^x \Rightarrow f'(x) = e^x$ $f(x) = \ln x \Rightarrow f'(x) = \frac{1}{x}$ $y = g(u), \text{ where } u = f(x) \Rightarrow \frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$ $y = uv \Rightarrow \frac{dy}{dx} = u \frac{dv}{dx} + v \frac{du}{dx}$ $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{v^2}$ |
| <b>SL 5.9</b> | <p>Acceleration</p> <p>Distance travelled from <math>t_1</math> to <math>t_2</math></p> <p>Displacement from <math>t_1</math> to <math>t_2</math></p>                                                                      | $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ $\text{distance} = \int_{t_1}^{t_2}  v(t)  dt$ $\text{displacement} = \int_{t_1}^{t_2} v(t) dt$                                                                                                                                                                                                                                                                                                          |

|                    |                                                  |                                                                                                                         |
|--------------------|--------------------------------------------------|-------------------------------------------------------------------------------------------------------------------------|
| <b>SL<br/>5.10</b> | Standard integrals                               | $\int \frac{1}{x} dx = \ln x  + C$ $\int \sin x dx = -\cos x + C$ $\int \cos x dx = \sin x + C$ $\int e^x dx = e^x + C$ |
| <b>SL<br/>5.11</b> | Area of region enclosed by a curve and $x$ -axis | $A = \int_a^b  y  dx$                                                                                                   |

## Topic 5: Calculus – HL only

|                     |                                            |                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                                          |
|---------------------|--------------------------------------------|------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>AHL<br/>5.12</b> | Derivative of $f(x)$ from first principles | $y = f(x) \Rightarrow \frac{dy}{dx} = f'(x) = \lim_{h \rightarrow 0} \left( \frac{f(x+h) - f(x)}{h} \right)$                                                                                                                                                                                                                                                                                                                                                                                                                             |
| <b>AHL<br/>5.15</b> | Standard derivatives                       | $f(x) = \tan x \Rightarrow f'(x) = \sec^2 x$ $f(x) = \sec x \Rightarrow f'(x) = \sec x \tan x$ $f(x) = \operatorname{cosec} x \Rightarrow f'(x) = -\operatorname{cosec} x \cot x$ $f(x) = \cot x \Rightarrow f'(x) = -\operatorname{cosec}^2 x$ $f(x) = a^x \Rightarrow f'(x) = a^x (\ln a)$ $f(x) = \log_a x \Rightarrow f'(x) = \frac{1}{x \ln a}$ $f(x) = \arcsin x \Rightarrow f'(x) = \frac{1}{\sqrt{1-x^2}}$ $f(x) = \arccos x \Rightarrow f'(x) = -\frac{1}{\sqrt{1-x^2}}$ $f(x) = \arctan x \Rightarrow f'(x) = \frac{1}{1+x^2}$ |

|                     |                                                                                                        |                                                                                                                                                                                                                                                                                                                             |
|---------------------|--------------------------------------------------------------------------------------------------------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| <b>AHL<br/>5.15</b> | Standard integrals                                                                                     | $\int a^x dx = \frac{1}{\ln a} a^x + C$ $\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \arctan\left(\frac{x}{a}\right) + C$ $\int \frac{1}{\sqrt{a^2 - x^2}} dx = \arcsin\left(\frac{x}{a}\right) + C, \quad  x  < a$                                                                                                           |
| <b>AHL<br/>5.16</b> | Integration by parts                                                                                   | $\int u \frac{dv}{dx} dx = uv - \int v \frac{du}{dx} dx \quad \text{or} \quad \int u dv = uv - \int v du$                                                                                                                                                                                                                   |
| <b>AHL<br/>5.17</b> | <p>Area of region enclosed by a curve and y-axis</p> <p>Volume of revolution about the x or y-axes</p> | $A = \int_a^b  x  dy$ $V = \int_a^b \pi y^2 dx \quad \text{or} \quad V = \int_a^b \pi x^2 dy$                                                                                                                                                                                                                               |
| <b>AHL<br/>5.18</b> | <p>Euler's method</p> <p>Integrating factor for <math>y' + P(x)y = Q(x)</math></p>                     | <p><math>y_{n+1} = y_n + h \times f(x_n, y_n); \quad x_{n+1} = x_n + h</math>, where <math>h</math> is a constant (step length)</p> $e^{\int P(x) dx}$                                                                                                                                                                      |
| <b>AHL<br/>5.19</b> | <p>Maclaurin series</p> <p>Maclaurin series for special functions</p>                                  | $f(x) = f(0) + x f'(0) + \frac{x^2}{2!} f''(0) + \dots$ $e^x = 1 + x + \frac{x^2}{2!} + \dots$ $\ln(1+x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \dots$ $\sin x = x - \frac{x^3}{3!} + \frac{x^5}{5!} - \dots$ $\cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \dots$ $\arctan x = x - \frac{x^3}{3} + \frac{x^5}{5} - \dots$ |